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Discrete D-branes in AdS_3 and in the 2d black hole

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Abstract: I show how the AdS_2 D-branes in the Euclidean AdS_3 string theory are related to the continuous D-branes in Liouville theory. I then propose new discrete D-branes in the Euclidean AdS_3 which correspond to the discrete D-branes in Liouville theory. These new D-branes satisfy the appropriate shift equations. They give rise to two families of discrete D-branes in the 2d black hole, which preserve different symmetries.

Keywords: Conformal Field Models in String Theory, D-branes.

 http://jhep.sissa.it/archive/papers/jhep082006015/jhep082006015.pdf

Contents

1. Introduction and overview

Liouville theory and string theories with an affine $\widehat{s\ell_2}$ symmetry have played an important rôle in recent studies of time-dependent string theory, two-dimensional quantum gravity, and the AdS/CFT correspondence. The features of these theories which are well-understood suggest that they share many important properties. This is not surprising considering that the Virasoro algebra of Liouville theory can be obtained from the $\widehat{s\ell_2}$ affine Lie algebra by a quantum Hamiltonian reduction [1]. This suggests that Liouville theory can be found as a subsector of theories with an $\widehat{s\ell_2}$ symmetry. For example, AdS_3 string theory can be reduced to Liouville theory via a topological twist [2, 3].

Conversely, it would be interesting to reconstruct the full AdS_3 string theory in terms of the better-understood Liouville theory. A hint that this can be done comes from Zamolodchikov and Fateev's relation [4] between the Knizhnik–Zamolodchikov (KZ) and Belavin–Polyakov–Zamolodchikov (BPZ) systems of differential equations, which reflect the $\widehat{s\ell_2}$ and Virasoro symmetries respectively. More recently, all correlation functions of the H_3^+ model (the Euclidean version of AdS_3 string theory) on a sphere have been written in terms of Liouville correlation functions [5]. Proving this relation relied on the prior knowledge of

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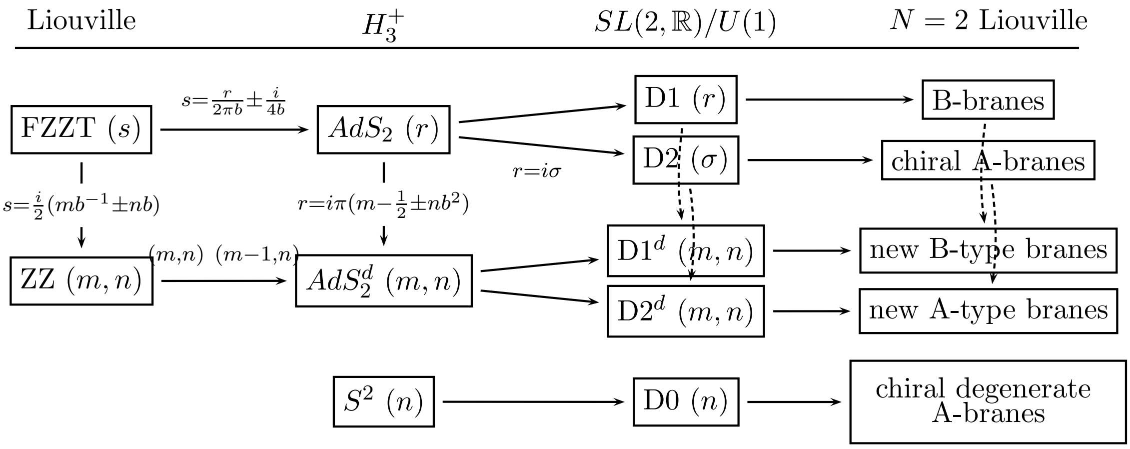
these H_3^+ correlation functions in terms of well-characterized objects, namely the three-point structure constants and the conformal blocks. However, the H_3^+-Liouville relation would be most useful if it allowed the construction of previously unknown objects in the H_3^+ model from known objects in Liouville theory. One purpose of this article is to demonstrate that it indeed does.

The new objects in the H_3^+ model which I plan to construct are discrete D-branes (in both meanings of having a discrete open string spectrum and coming in a discrete family) which correspond to the Zamolodchikov–Zamolodchikov (ZZ) D-branes in Liouville theory [6]. I will first determine a relation between the known continuous AdS_2 branes in the H_3^+ model [7] and the continuous Fateev–Zamolodchikov–Zamolodchikov–Teschner (FZZT) D-branes in Liouville theory [8, 9]. The main feature of this relation is the correspondence (2.8) between the parameters of these families of D-branes, which associates two different FZZT branes to one AdS_2 brane. Moreover, it is possible to relate the correlators of bulk fields in the presence of FZZT and AdS_2 branes eq. (2.10), but only in a particular regime which I will call the bulk regime. This is due to singularities in the H_3^+ conformal blocks, which have a clear interpretation – but so far no resolution – in terms of Liouville theory.

The relation between FZZT and AdS_2 branes will then suggest a natural ansatz for a family of discrete branes in H_3^+ parametrized by two integers (m, n), related to the ZZ branes of Liouville theory. The most useful characterization of these branes, which I will call AdS_2^d branes, is the relation to the AdS_2 branes eq. (3.4). (The name AdS_2^d refers to that relation and not to the geometry of the new discrete branes.) These AdS_2^d branes will be shown to be solutions of the same shift equation that was checked for the AdS_2 branes [7]. How to modify this equation for the case of discrete branes will be suggested by Liouville theory. I will then propose a tentative relation between $\widehat{s\ell_2}$ representations and D-branes in H_3^+, inspired by the Cardy relation which holds in rational conformal field theories, and which may help understand which H_3^+ D-branes can be related to Liouville branes and which ones cannot.

From the new discrete D-branes in the H_3^+ model, two families of compact D-branes in the 2d "cigar" black hole $SL(2, \mathbb{R})/U(1)$ obeying two different gluing conditions can be constructed along the lines of [10]. Some of these D-branes have a geometric interpretation as D0-branes at the tip of the cigar, the others do not have any geometric interpretation. These new D-branes in the 2d black hole can then easily be translated into D-branes in the $N = 2$ Liouville theory in Hosomichi's formalism [11], which provides a second independent shift equation.



2. AdS_2 D-branes from Liouville theory

The aim of this section is to generalize the relation between H_3^+ and Liouville bulk correlators on the Riemann sphere [5] to correlators of bulk fields in the presence of worldsheet boundaries described by continuous D-branes: the AdS_2 branes on the H_3^+ side, the FZZT branes on the Liouville side.

The relation between bulk correlators on the sphere can be decomposed into relations between bulk conformal blocks on the one hand, and bulk three-point structure constants on the other hand [12]. The introduction of a worldsheet boundary implies a modification of the conformal blocks, and the introduction of extra structure constants, namely the one-point functions (which must vanish when no boundary is there to break the worldsheet translation invariance).

Let me briefly recall that Liouville theory is a two-dimensional conformal field theory on a worldsheet parametrized by a complex number z. The theory may be defined in terms of a field $\phi(z)$ by the action:

$$S^{\text{Liouville}} = \int d^2z \, \left(|\partial_z \phi|^2 + \mu_L e^{2b\phi} \right) \, . \tag{2.1}$$

The H_3^+ model describes strings in a three-dimensional space and therefore requires three fields $\phi, \gamma, \bar{\gamma}$:

$$S^{H_3^+} = k \int d^2z \, \left(|\partial_z \phi|^2 + e^{2\phi} \partial\gamma \bar{\partial}\bar{\gamma} \right) \, . \tag{2.2}$$

A more complete review with relevant references can be found in [5].

2.1 Comparison of one-point functions

Consider one-point functions of the closed string worldsheet fields $V_\alpha(z)$ in Liouville theory and $\Phi^j(x|z)$ in the H_3^+ model. From the bulk H_3^+-Liouville relation, the Liouville momentum α and the H_3^+ spin j are related by:

$$\alpha = b(j+1) + \frac{1}{2b}, \tag{2.3}$$

where the Liouville parameter b is related to the H_3^+ model level k by $b^2 = \frac{1}{k-2}$. In terms of j, the one-point function for the Liouville FZZT brane parametrized by the real number s is [8, 9]

$$\left\langle V_{\alpha=b(j+1)+\frac{1}{2b}}(z) \right\rangle_s = \frac{\Psi_s^{\rm FZZT}}{|z-\bar{z}|^{2\Delta_\alpha}},$$
$$\Psi_s^{\rm FZZT} = (\pi\mu_L\gamma(b^2))^{-j-\frac{1}{2}} \frac{1}{\pi 2^{\frac{1}{4}} b} \Gamma(2j+1)\Gamma(1+b^2(2j+1)) \cosh 2\pi bs(2j+1), \quad (2.4)$$

where z is the complex worldsheet coordinate and μ_L the Liouville interaction strength. The one-point function for an AdS_2 brane in H_3^+ with real parameter r is: [7, 13]

$$\Psi_r^{AdS_2}(x) = \nu_b^{j+\frac{1}{2}}(8b^2)^{-\frac{1}{4}}|x+\bar{x}|^{2j}\Gamma(1+b^2(2j+1))e^{-r(2j+1){\rm sgn}(x+\bar{x})}, \quad (2.5)$$

where $\nu_b = \pi\frac{\Gamma(1-b^2)}{\Gamma(1+b^2)}$ so that $\Phi^{j=0}$ is the identity field (in slight contrast to [7]), and x is a complex isospin variable which labels states within a continuous $SL(2,\mathbb{C})$ representation of spin j.

This one-point function of the x-basis fields is written here for later use, but is not clearly related to the one-point function in Liouville theory. Instead, the bulk H_3^+-Liouville relation suggests to consider the μ-basis fields

$$\Phi^j(\mu|z) = |\mu|^{2j+2}\int_{\mathbb{C}} d^2x\ e^{\mu x-\bar{\mu}\bar{x}}\Phi^j(x|z), \quad (2.6)$$

whose one-point functions are obtained from eq. (2.5) after a straightforward calculation:

$$\left\langle \Phi^j(\mu|z) \right\rangle_r = \frac{\Psi_r^{AdS_2}}{|z-\bar{z}|^{2\Delta_j}},$$
$$\Psi_r^{AdS_2} = |\mu|\delta(\Re\mu)\nu_b^{j+\frac{1}{2}}\pi(8b^2)^{-\frac{1}{4}}\Gamma(2j+1)\Gamma(1+b^2(2j+1))\cosh(2j+1)(r-i\frac{\pi}{2}{\rm sgn}\Im\mu). \quad (2.7)$$

It is now obvious that the AdS_2 D-brane one-point function is essentially the same as that of an FZZT brane (2.4), but depending on ${\rm sgn}\Im\mu$ two different boundary parameters may appear:

$$s_\pm = \frac{r}{2\pi b} \pm \frac{i}{4b}. \quad (2.8)$$

Such a relation could have been expected on several grounds. First, the FZZT branes are invariant under $s \to -s$ whereas the AdS_2 branes are not invariant under $r \to -r$, so there cannot be a one-to-one relation between the parameters r and s. Second, the $SL(2,\mathbb{R})$ symmetry of the AdS_2 brane, which acts on the x parameter, does not completely determine the x dependence of the one-point function, but allows an arbitrary dependence on ${\rm sgn}(x+\bar{x})$ [7]. Therefore the one-point function for an AdS_2 brane involves two structure constants (instead of one in Liouville theory), which in the μ basis are encoded in the ${\rm sgn}\Im\mu$ dependence. Third, the difference $s_+ - s_- = \frac{i}{2b}$ is the jump in Liouville boundary condition induced by a boundary degenerate field $B_{-\frac{1}{2b}}$. This is not surprising in view of the appearance of such degenerate fields in the H_3^+-Liouville relation beyond the one-point function discussed below.

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2.2 Comparison of conformal blocks

The H_3^+ bulk conformal blocks are controlled by the Knizhnik–Zamolodchikov equations [14], which are enough to determine their relation with Liouville conformal blocks [12]. Let me determine the KZ equations satisfied by the conformal blocks involved in the correlator of n bulk fields in the presence of an AdS_2 brane $\left\langle \Phi^{j_1}(\mu_1|z_1)\cdots\Phi^{j_n}(\mu_n|z_n)\right\rangle_r$. In Wess–Zumino–Witten models with symmetry-preserving boundary conditions, such KZ equations are identical to the KZ equations satisfied by a correlator of $2n$ bulk fields on the sphere (at points $z_1,\cdots z_n,\ \bar{z}_1\cdots\bar{z}_n$), modulo a twist of the currents acting on the reflected fields if the gluing conditions are non-trivial. In the case of AdS_2 branes, the gluing conditions are trivial as I will now show.

Let me call $J^a(z), \bar{J}^a(\bar{z})$ the left- and right-moving currents of the H_3^+ model [15]. Their modes generate an $\widehat{s\ell_2}(\mathbb{C})\times\widehat{s\ell_2}(\mathbb{C})$ affine Lie algebra. Their zero modes act on the fields $\Phi^j(x|z)$ or $\Phi^j(\mu|z)$ as differential operators with respect to the isospin variables x or μ:

$$\begin{aligned} J_0^- &= \frac{\partial}{\partial x} &&= \mu\,, \\ J_0^0 &= x\frac{\partial}{\partial x}-j &&= -\mu\frac{\partial}{\partial\mu}\,, \\ J_0^+ &= x^2\frac{\partial}{\partial x}-2jx &&= \mu\frac{\partial^2}{\partial\mu^2}-\frac{j(j+1)}{\mu}\,, \end{aligned} \tag{2.9}$$

and the currents $\bar{J}_0^a$ are defined by repacing x,μ with $\bar{x},\bar{\mu}$. Note that this definition of the $\bar{J}_0^a$ currents is incompatible with the change of basis (2.6) and is therefore basis-dependent. As a result, the gluing conditions will also be basis-dependent.

The μ-basis one-point function of the AdS_2 brane satisfies $(J_0^a+\bar{J}_0^a)\Psi_r^{AdS_2}(\mu)=0$, which corresponds to the trivial gluing condition $J=\bar{J}$ (see for instance [16]). Thus, it satisfies the same KZ equations as the bulk two-point function $\left\langle\Phi^j(\mu|z)\Phi^j(\bar{\mu}|\bar{z})\right\rangle$. Indeed, the μ-dependences are similar: $|\mu|^2\delta^{(2)}(\mu+\bar{\mu})$ for the bulk two-point function, $\mu\delta(\mu+\bar{\mu})$ for the one-point function. In contrast, the x-basis one-point function has an $|x+\bar{x}|^{2j}$ factor which contrasts with the bulk two-point function $|x-\bar{x}|^{4j}$. This reflects the fact that the gluing conditions are non-trivial in the x-basis.

Since the correlator $\left\langle \Phi^{j_1}(\mu_1|z_1)\cdots\Phi^{j_n}(\mu_n|z_n)\right\rangle_r$ satisfies the same KZ equations as a bulk correlator with $2n$ fields, these equations are equivalent to BPZ equations via Sklyanin's separation of variables, as explained in [5, 17]. This leads to the following relation between H_3^+ and Liouville correlators in the presence of worldsheet boundaries, where the equality so far means "satisfies the same differential equations as":

$$\begin{aligned} \left\langle\prod_{\ell=1}^n \Phi^{j_\ell}(\mu_\ell|z_\ell)\right\rangle_r &= \pi^2\sqrt{\frac{b}{2}}(-1)^n\left|\textstyle\sum_{i=1}^n\Re(\mu_i z_i)\right|\delta\left(\Re(\textstyle\sum_{i=1}^n\mu_i)\right) \\ &\quad\times|\Theta_{2n}|^{\frac{k-2}{2}}\left\langle\prod_{\ell=1}^n V_{\alpha_\ell}(z_\ell)\prod_{a=1}^{n-1}V_{-\frac{1}{2b}}(y_a)\right\rangle_{s=\frac{r}{2\pi b}-\frac{i}{4b}\mathrm{sgn}\sum_{i=1}^n\Im\mu_i}\,, \end{aligned} \tag{2.10}$$

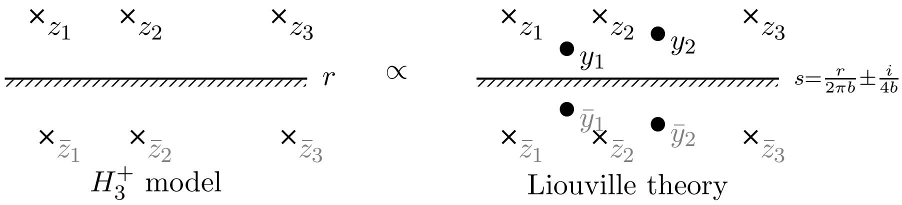
In this equation the following conventions are used: the momenta and spins are related as in eq. (2.3), I assume $\mu_L = \frac{b^2}{\pi^2}$, the function Θ_{2n} is defined by

$$\Theta_{2n} = \frac{\prod_{\ell<\ell'\leq n} |z_{\ell\ell'}|^2 \prod_{\ell,\ell'\leq n} (z_\ell - \bar{z}_{\ell'}) \prod_{a<a'\leq n-1} |y_{aa'}|^2 \prod_{a,a'\leq n-1} (y_a - \bar{y}_{a'})}{\prod_{\ell=1}^{n} \prod_{a=1}^{n-1} |z_\ell - y_a|^2 |z_\ell - \bar{y}_a|^2}, \tag{2.11}$$

and most importantly the y_a are the roots with positive imaginary parts of the real polynomial $P(t)$ defined by:

$$\sum_{\ell=1}^{n} \left(\frac{\mu_\ell}{t-z_\ell} + \frac{\bar{\mu}_\ell}{t-\bar{z}_\ell} \right) = \left[\sum_{\ell=1}^{n} (\mu_\ell z_\ell + \bar{\mu}_\ell \bar{z}_\ell) \right] \frac{P(t)}{\prod_{\ell=1}^{n} (t-z_\ell)(t-\bar{z}_\ell)}. \tag{2.12}$$

In the case $n = 3$, the equation (2.10) can be represented as:



(2.13)

The reflected fields at $(\bar{z}_1 \cdots \bar{z}_n)$ in the lower half-plane are not physical, but they are indicated in this picture because they appear in the KZ or BPZ equations satisfied by the physical correlators of eq. (2.10).

In this subsection I only argued that both sides of equation (2.10) satisfy identical systems of differential equations. This amounts to a relation between the conformal blocks from which the correlators are built. In the next subsection I will complete the argument for equation (2.10) and show that it holds in a certain regime.

2.3 The bulk regime

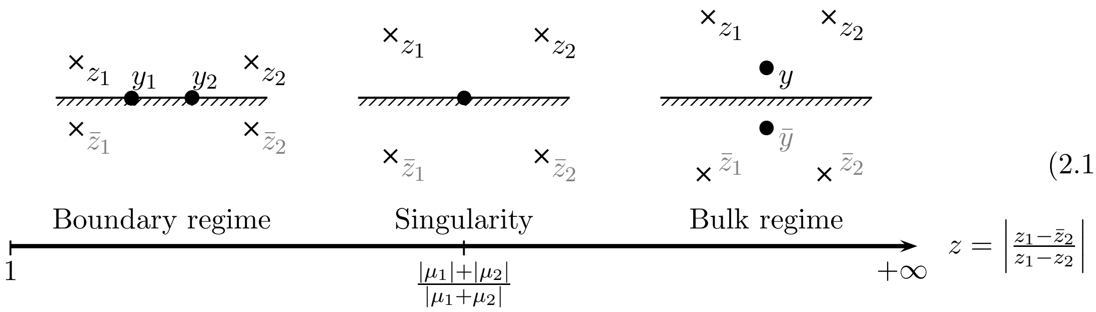
From the explicit expressions for the one-point functions (2.4), (2.7) it is easy to check that the equation (2.10) holds in the case $n = 1$, which does not involve any insertion of degenerate Liouville fields $V_{-\frac{1}{2b}}$. One could then think that it is possible to prove equation (2.10) by a recursion on n, using the bulk operator product expansion to reduce the case of the n-point function in the limit $z_1 \to z_2$ to the case of the $n-1$-point function. (The bulk OPEs in the H_3^+ model and Liouville theory are indeed related in a way which would suit such an argument [5].) Then one would rely on the KZ equation to extend the relation (2.10) to all values of z_i, away from the limit $z_1 \to z_2$.

However, this argument does not work because the conformal blocks which solve the KZ equations have singularities. These singularities are most easily seen in the corresponding Liouville theory conformal blocks: they occur whenever one of the y_a becomes real. Indeed the y_a are defined as the roots of the real polynomial $P(t)$ (2.12). Such a polynomial can have real roots and pairs of complex conjugate roots. Let me call the *bulk regime* the range of values of μ_ℓ, z_ℓ such that all the roots of $P(t)$ are complex. The repeated use of the bulk OPE $z_1 \to z_2 \to \cdots z_n$ (as required by the recursion above) is possible only in the bulk regime, because the definition of $P(t)$ (2.12) implies that for $z_1 \to z_2$ some root y_1 of $P(t)$ will also move close to z_1, and therefore in the bulk. Thus, the equation (2.10) holds only in the bulk regime. Unfortunately, this prevents the easy determination of an H_3^+-Liouville relation in other bases like the x basis, which would involve an integration over all values of μ_ℓ.

Let me illustrate the singularities of the conformal blocks in the case of a two-point function $\left\langle \Phi^{j_1}(\mu_1|z_1)\Phi^{j_2}(\mu_2|z_2)\right\rangle_r$. In this case the polynomial $P(t)$ has degree two and its roots are complex provided

$$z \equiv \left|\frac{z_1-\bar{z}_2}{z_1-z_2}\right| > \frac{|\mu_1|+|\mu_2|}{|\mu_1+\mu_2|} \ . \tag{2.14}$$

The cross-ratio z varies from 1 when the two H_3^+ bulk fields are far apart or close to the boundary, to $+\infty$ when they are close together or far from the boundary. The corresponding Liouville configurations are:



(2.15)

In the boundary regime, the relation between the KZ and BPZ equations still holds. However it is not clear that a relation between H_3^+ and Liouville correlators can be found. Such a relation would have to specify which boundary parameters appear in Liouville theory. The boundary degenerate fields induce jumps of the boundary parameter s by the quantity $\frac{i}{2b}$ [8]. The fact that the two boundary parameters $s_\pm$ (2.8) differ by this quantity is very suggestive, but more work needs to be done. This issue is however not relevant to the present article, whose purpose is to find new discrete D-branes in the H_3^+ model.

3. More branes in the Euclidean AdS_3

In this section I will show that the relation between Liouville FZZT branes and AdS_2 branes in the H_3^+ model suggests a natural ansatz for new discrete D-branes in the H_3^+ model, which will be related to the discrete ZZ-branes in Liouville theory. This ansatz will then be subjected to a number of tests.

Let me first briefly review the ZZ branes and their relation to the continuous FZZT branes. The ZZ branes are parametrized by two strictly positive integers (m,n) and are described by the one-point functions [6]

$$\begin{aligned}&\left\langle V_{\alpha=b(j+1)+\frac{1}{2b}}(z)\right\rangle_{(m,n)} = \frac{\Psi^{\rm ZZ}_{(m,n)}}{|z-\bar{z}|^{2\Delta_\alpha}},\\ &\Psi^{\rm ZZ}_{(m,n)} = (\pi\mu_L\gamma(b^2))^{-j-\frac{1}{2}}\frac{2^{\frac{3}{4}}}{\pi b}\Gamma(2j+1)\Gamma(1+b^2(2j+1))\sin\pi m(2j+1)\sin\pi n b^2(2j+1)\ .\end{aligned} \tag{3.1}$$

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A well-known property of these ZZ branes which will be most useful in the following is:

$$\Psi^{\rm ZZ}_{(m,n)} = \Psi^{\rm FZZT}_{\frac{i}{2}(mb^{-1}+nb)} - \Psi^{\rm FZZT}_{\frac{i}{2}(mb^{-1}-nb)}. \tag{3.2}$$

3.1 An ansatz for new discrete D-branes

The previous section demonstrated that an AdS_2 brane with boundary parameter r is related to FZZT branes with boundary parameters $s = \frac{r}{2\pi b} - \frac{i}{4b}\mathrm{sgn}\Im\mu$. It is natural to look for discrete branes in the H_3^+ model which would preserve the same symmetries as the AdS_2 branes (in other words, they would obey the same gluing conditions) and which would be related in a similar manner to ZZ branes with parameters depending on $\mathrm{sgn}\Im\mu$. In addition, I have explained that the difference of the two possible boundary parameters has an interpretation as the jump induced by a boundary degenerate field, which is quite natural considering the appearance of such fields in the boundary regime (2.15). Through the relation between ZZ and FZZT branes eq. (3.2), this jump corresponds to a jump $m \to m-1$ of the parameter m of the ZZ branes. This suggests the following relation:

New discrete brane in H_3^+	Liouville ZZ branes	
(m,n) strictly positive integers	$\begin{cases} (m-1,n) & \text{if } \mathrm{sgn}\Im\mu > 0 \\ (m,n) & \text{if } \mathrm{sgn}\Im\mu < 0 \end{cases}$	(3.3)

I will call "discrete AdS_2 branes" or "AdS_2^d branes" these new branes. Their above definition in terms of ZZ branes can be translated into a relation with AdS_2 branes via the ZZ-FZZT relation eq. (3.2) and the AdS_2-FZZT relation of the previous section:

$$\Psi^{AdS_2^d}_{(m,n)} = \Psi^{AdS_2}_{i\pi(m-\frac{1}{2}+nb^2)} - \Psi^{AdS_2}_{i\pi(m-\frac{1}{2}-nb^2)} . \tag{3.4}$$

The essential feature of this relation is the shift $-\frac{1}{2}$, which directly corresponds to the shift of the boundary parameters in the AdS_2-FZZT relation eq. (2.8). Let me write explicitly the one-point function of the AdS_2^d branes in the x basis:

$$\Psi^{AdS_2^d}_{(m,n)}(x) = \nu_b^{j+\frac{1}{2}}(8b^2)^{-\frac{1}{4}}|x+\bar{x}|^{2j}\Gamma(1+b^2(2j+1)) \\ \times 2i\mathrm{sgn}(x+\bar{x})\, e^{-i\pi(m-\frac{1}{2})(2j+1)\mathrm{sgn}(x+\bar{x})} \sin \pi b^2 n(2j+1) . \tag{3.5}$$

Naturally, the relation (3.4) provides a simple way to derive the one-point functions of the AdS_2^d branes in any basis. I have used the x basis because the shift equation of the next subsection is formulated in this basis.

3.2 Verification of the shift equation

The one-point function for an AdS_2 brane was found in [7] by solving a shift equation indicating how it should behave under shifts $j \to j \pm \frac{1}{2}$. A modified version of the shift equation is expected to hold for discrete branes preserving the same symmetries. This expectation is based on the study of shift equations for ZZ and FZZT branes in Liouville theory [8, 6] which I now review.

The shift equations for ZZ and FZZT branes are of the type:

$$R^a_s \Psi^a_s(\alpha) = F_- \Psi^a_s(\alpha - \frac{b}{2}) + F_+ \Psi^a_s(\alpha + \frac{b}{2}), \tag{3.6}$$

where the index a means ZZ or FZZT, with brane parameters generically called s. The coefficients $F_\pm$ on the right-hand side do not depend on the type or parameter of the D-brane because they are fusing matrix elements. However, the quantity R^a_s depends on the type of brane: [1]

$$R^{\mathrm{FZZT}}_s = R^{\mathrm{FZZT}}(-\frac{b}{2}, Q|s) = -2\pi \sqrt{\frac{\mu_L}{\sin \pi b^2}} \frac{\Gamma(-1-2b^2)}{\Gamma(-b^2)^2} \cosh 2\pi b s\,, \tag{3.7}$$

$$R^{\mathrm{ZZ}}_{(m,n)} = \frac{\Psi^{\mathrm{ZZ}}_{(m,n)}(\alpha = -\frac{b}{2})}{\Psi^{\mathrm{ZZ}}_{(m,n)}(\alpha = 0)}\,, \tag{3.8}$$

where the bulk-boundary structure constant value $R^{\mathrm{FZZT}}(-\frac{b}{2}, Q|s)$ was derived in [8] by a free field computation and can also be deduced from the general formula for the bulk-boundary structure constant [18] by carefully taking the relevant limit as sketched in [19].

One may wonder how the shift equations (3.6), where the factor R^a_s depends on the type of brane ($a \in \{\mathrm{ZZ}, \mathrm{FZZT}\}$), can be compatible with the linear relation (3.2) between ZZ and FZZT branes. The compatibility actually requires the non-trivial relation $R^{\mathrm{ZZ}}_{(m,n)} = R^{\mathrm{FZZT}}_{s=i\frac{mb^{-1}+nb}{2}} = R^{\mathrm{FZZT}}_{s=i\frac{mb^{-1}-nb}{2}}$. A direct computation shows that this relation is indeed obeyed:

$$\frac{\Psi^{\mathrm{ZZ}}_{(m,n)}(-\frac{b}{2})}{\Psi^{\mathrm{ZZ}}_{(m,n)}(0)} = R^{\mathrm{FZZT}}_{i\frac{mb^{-1}+nb}{2}} = R^{\mathrm{FZZT}}_{i\frac{mb^{-1}-nb}{2}} = -2\pi \sqrt{\frac{\mu_L}{\sin \pi b^2}} \frac{\Gamma(-1-2b^2)}{\Gamma(-b^2)^2} (-1)^m \cos \pi n b^2. \tag{3.9}$$

This analysis of the Liouville branes' shift equations can be generalized to H_3^+ branes' shift equations. The continuous AdS_2 branes are indeed known to satisfy an equation of the type [7]

$$R^{AdS_2}_r \Psi^{AdS_2}_r(j) = F^{H_3^+}_- \Psi^{AdS_2}_r(j - \frac{1}{2}) + F^{H_3^+}_+ \Psi^{AdS_2}_r(j + \frac{1}{2}). \tag{3.10}$$

In the notations of [7], the quantity $R^{AdS_2}_r$ can be computed explicitly as $R^{AdS_2}_r = (x + \bar{x}) B(\frac{1}{2}) A(\frac{1}{2}, 0|r)$ (see in particular the equation (3.28) therein) [2].

Now the shift equation for discrete branes in H_3^+ should be identical to that for continuous branes, except for the replacement of $R^{AdS_2}_r$ with

$$R^{AdS_2^d}_{(m,n)} = \frac{\Psi^{AdS_2^d}_{(m,n)}(j = \frac{1}{2})}{\Psi^{AdS_2^d}_{(m,n)}(j = 0)}. \tag{3.11}$$

[1] In the article [6] the denominator $\Psi^{\mathrm{ZZ}}(\alpha = 0)$ is absent from $R^{\mathrm{ZZ}}_{(m,n)}$ because the one-point function is normalized so that $\Psi^{\mathrm{ZZ}}(\alpha = 0) = 1$.

[2] Note that $\nu_b = \pi \frac{\Gamma(1-b^2)}{\Gamma(1+b^2)}$ now has an extra factor π wrt [7] so that $\Phi^{j=0}$ is the identity field, see eq. (2.5) of the present article and footnote 7 of [7]. Also note that the requirement $B(j = 0) = 1$ leads to a different sign for $B(j)$ as compared to [7].

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Does the ansatz (3.4) satisfy the resulting shift equation? Like in Liouville theory, the shift equation for discrete branes boils down to the equations

$$R^{AdS_2^d}_{(m,n)} \stackrel{!}{=} R^{AdS_2}_{r=i\pi(m-\frac{1}{2}+nb^2)} \stackrel{!}{=} R^{AdS_2}_{r=i\pi(m-\frac{1}{2}-nb^2)}. \tag{3.12}$$

These equations can now be checked by direct calculation, and the three quantities to be compared are indeed all equal to

$$2i|x+\bar{x}|\mathrm{sgn}(x+\bar{x})\sqrt{\nu_b}\frac{\Gamma(1+2b^2)}{\Gamma(1+b^2)}(-1)^m\cos\pi nb^2\ . \tag{3.13}$$

3.3 Checks and interpretations *à la Cardy*

3.3.1 D-branes and representation theory

Let me discuss how the proposed discrete AdS_2 branes help to complete the list of D-branes in the Euclidean AdS_3. Cardy has shown that in rational two-dimensional conformal field theories, symmetry-preserving D-branes are naturally associated to representations of the relevant symmetry algebra [20]. This idea can be extended to Liouville theory. To start with, the continuous FZZT branes are naturally associated to the continuous representations of the Virasora algebra, which appear in the physical Liouville spectrum and have momenta $\alpha \in \frac{Q}{2}+i\mathbb{R}$ (with $Q=b+b^{-1}$). In order to account for the ZZ branes in terms of representation theory, one has to go beyond the physical spectrum and take into account the degenerate representations appearing in the Kac table, with momenta

$$2\alpha_{mn}-Q=mb^{-1}+nb\,, \tag{3.14}$$

where (m,n) are still strictly positive integers, and I ignore the reflected degenerate representations $2\alpha_{mn}-Q=-(mb^{-1}+nb)$ because the reflection symmetry of Liouville theory makes them redundant. Now, the relation (2.3) between the Liouville momentum and the H_3^+ spin relates the Virasoro degenerate representations to $\widehat{s\ell_2}$ degenerate representations with spins

$$2j_{mn}+1=mb^{-2}+n\ . \tag{3.15}$$

The discrete AdS_2 branes should be considered as associated with such representations, whereas the ordinary AdS_2 branes would be associated with the physical continuous representations $j\in -\frac{1}{2}+i\mathbb{R}$. This interpretation of the AdS_2 branes was already considered in [7] (section 4.2), which suggested the following relation between representation spins j and brane parameters r:

$$j(r)=-\frac{1}{2}-\frac{1}{4b^2}+i\frac{r}{2\pi b^2}\ . \tag{3.16}$$

However, as observed in [7], this relation does not give physical values $j\in-\frac{1}{2}+i\mathbb{R}$ for r real due to the term $-\frac{1}{4b^2}$. But this term precisely corresponds to the shift in the AdS_2-FZZT relation (2.8), and now seems rather natural. The reflection symmetry of the spectrum $j\to -j-1$ then corresponds to the invariance of the FZZT branes under $s\to -s$.

Now replacing r in eq. (3.16) with the values appropriate for discrete AdS_2 branes (3.4) gives the spins of the $\widehat{s\ell_2}$ degenerate representations with null vector at nonzero level: $2j(r = i\pi[m - \frac{1}{2} + nb^2]) + 1 = -(mb^{-2} + n)$.

There is another series of degenerate representations of $\widehat{s\ell_2}$ with $m = 0$, which do not correspond to Virasoro degenerate representations because they have a null vector at level zero [21]. D-branes corresponding to these representations are therefore not expected to be simply related to Liouville theory objects. There exist natural candidates for such D-branes: the S^2 branes with imaginary radius of [7]. In contrast to the AdS_2 branes which preserve an $SL(2,\mathbb{R})$ symmetry out of the $SL(2,\mathbb{C})$ of the H_3^+ model, the S^2 branes preserve an SU(2) symmetry. The degenerate representations with level zero null vectors are unitary as SU(2) representations, and they indeed appear in the physical spectrum of the S^2 branes.

The representations mentioned so far are summarized in the following table, which should be compared to the picture of the moduli spaces of D-branes in H_3^+ and Liouville theory in the Introduction:

Virasoro	$\widehat{s\ell_2}$
$\alpha \in \frac{Q}{2} + i\mathbb{R}$	$j \in -\frac{1}{2} + i\mathbb{R}$
$2\alpha_{mn} - Q = mb^{-1} + nb$	$2j_{mn} + 1 = mb^{-2} + n$
	$2j_n + 1 = n$

3.3.2 Computation of the annulus amplitudes

In the context of rational conformal field theories, Cardy has shown that the consistency of the spectrum of open strings on a D-brane (i.e. the requirement that it consists of finitely many representations with positive integer multiplicities) leads to a strong constraint on the one-point function of that D-brane [20]. In non rational conformal field theories, the spectrum of open strings should consist of continous states with a positive density and/or discrete states with positive integer multiplicities. The consistency of the AdS_2 branes has already been checked in this way in [7, 22]. The study of this type of consistency conditions is sometimes called the *modular bootstrap* approach [23].

The open-string spectrum is related to the one-point function via the annulus amplitude $Z^{AdS_2^d}_{(m_1,n_1)(m_2,n_2)} = \mathrm{Tr}\tilde{q}^{L_0 - \frac{c}{24}}$ where the powers of $\tilde{q}$ are the energies of the open-string states [3]. Like the annulus amplitude for AdS_2 branes, the annulus amplitude for open strings stretched between two AdS_2^d branes is most easily computed in the μ basis. (A naive x-basis computation would give a wrong result due to an improper treatment of the

[3]With standard conventions: $\tilde{q} = \exp -\frac{2\pi i}{\tau}$ and $q = \exp 2\pi i\tau$ where τ is the modular parameter of the annulus

divergences [7].)

$$Z^{AdS_2^d}_{(m_1,n_1)(m_2,n_2)} = \int_{-\frac{1}{2}+i\mathbb{R}} dj \int_{\mathbb{C}} \frac{d^2\mu}{|\mu|^2} \Psi^{AdS_2^d}_{(m_1,n_1)} \left(\Psi^{AdS_2^d}_{(m_2,n_2)}\right)^* \frac{q^{-\frac{b^2}{4}(2j+1)^2}}{\prod_{\ell=1}^\infty (1-q^\ell)^3} \tag{3.17}$$

$$= \delta(0) \int_0^\infty 1 \times \sum_{n\in n_1\times n_2} \left(\sum_{m\in m_1\times m_2} + \sum_{m\in(m_1-1)\times(m_2-1)} \right) \chi_{mn}(\tilde{q}) \tag{3.18}$$

In this formula, $m\in m_1\times m_2$ means $|m_1-m_2|<m<m_1+m_2$ while m_1+m_2-m is an odd integer (like in $\frac{\sin m_1 x \sin m_2 x}{\sin x} = \sum_{m\in m_1\times m_2} \sin mx$), and $\chi_{mn}(q) = \eta^{-3}(q)(q^{-\frac{1}{4}(mb^{-1}+nb)^2} - q^{-\frac{1}{4}(mb^{-1}-nb)^2})$ is an $\widehat{s\ell_2}$ degenerate character [21]. The infinite prefactors (which come from the integral $\int_{\mathbb{C}} d^2\mu$) result from the $SL(2,\mathbb{R})$ symmetry of the AdS_2^d branes and are similar to infinite prefactors appearing in the annulus amplitude of AdS_2 branes [7]. In the case of AdS_2 branes, there was an extra divergence of the integral $\int dj$ at $j=-\frac{1}{2}$. This zero radial momentum divergence reflected the infinite extension of the AdS_2 branes in the radial direction and is absent in the case of the AdS_2^d branes.

Therefore, the spectrum of open strings on the AdS_2^d branes is consistent. The spectrum of open strings between AdS_2 and AdS_2^d branes is also made of discrete states with integer multiplicities, but these states can have imaginary conformal dimensions, as is clear from the formula:

$$Z^{AdS_2-AdS_2^d}_{r,(m,n)} \propto \int dj\, \frac{q^{-\frac{b^2}{4}(2j+1)^2}}{\prod_{\ell=1}^\infty(1-q^\ell)^3} \frac{\sin \pi n b^2(2j+1)}{\sin \pi b^2(2j+1)}$$
$$\times \left[\frac{\sin\pi(m-1)(2j+1)}{\sin\pi(2j+1)} \cosh(r-i\frac{\pi}{2})(2j+1) + \frac{\sin\pi m(2j+1)}{\sin\pi(2j+1)} \cosh(r+i\frac{\pi}{2})(2j+1) \right] . \tag{3.19}$$

The Gaussian integral on j will indeed yield powers of $\tilde{q}$ which are not real. In such cases I will say that $Z^{AdS_2-AdS_2^d}_{r\neq 0,(m,n)}$ has an *imaginary spectrum pathology.* Note however that this pathology is not an inconsistency of the conformal field theory with boundary conditions defined by AdS_2^d branes. The pathology only prevents the AdS_2^d branes to be interpreted as physical string theory objects in the presence of AdS_2 branes.

Actually, the ZZ branes with $(m,n)\neq(1,1)$ in Liouville also have this imaginary spectrum pathology, which does not prevent them from playing an important rôle in the theory. Note also that the pathology can be absent in the case of some branes constructed from the AdS_2^d branes as I will argue in the context of the 2d black hole $SL(2,\mathbb{R})/U(1)$.

4. More branes in the 2d black hole

D-branes in the 2d "cigar" Euclidean black hole $SL(2,\mathbb{R})/U(1)$ can be obtained from D-branes in the Euclidean AdS_3 by a descent procedure [10]. On the one hand this will yield more consistency checks for the new D-branes constructed in the present article, and on the other hand this will suggest a comparison with matrix model results.

4.1 Known branes and new branes in the 2d black hole

Let me now recall the one-point functions of the $SL(2,\mathbb{R})/U(1)$ bulk fields $\Phi^j_{n',w}$ in the presence of boundary conditions defined by the D-branes descending from S^2 and AdS_2 branes in H_3^+.

A D0-brane in the cigar descends from an S^2 branes in H_3^+ labelled by a strictly positive integer n:

$$\Psi_n^{D0} = \delta_{n'0}\nu_b^{j+\frac{1}{2}}\frac{k^{\frac{1}{4}}b^{-\frac{1}{2}}}{2\pi(-1)^{nw+1}}\frac{\Gamma(\frac{kw}{2}-j)\Gamma(-\frac{kw}{2}-j)}{\Gamma(-2j)}\Gamma(1+b^2(2j+1))\sin\pi nb^2(2j+1) \quad (4.1)$$

A D1-brane in the cigar descends from an AdS_2 brane in H_3^+ with a real parameter r and an angle θ_0:

$$\Psi_r^{D1} = \delta_{w,0}e^{in'\theta_0}\nu_b^{j+\frac{1}{2}}\frac{k^{-\frac{1}{4}}b^{-\frac{1}{2}}}{2}\frac{\Gamma(2j+1)\Gamma(1+b^2(2j+1))}{\Gamma(1+j+\frac{n'}{2})\Gamma(1+j-\frac{n'}{2})}\left(e^{-r(2j+1)}+(-1)^{n'}e^{r(2j+1)}\right) \quad (4.2)$$

A D2-brane in the cigar also descends from an AdS_2 brane in H_3^+, whose parameter r now has to be taken pure imaginary $r=i\sigma$. The real parameter σ of the D2-branes is quantized in units of $2\pi b^2$ and bounded $|\sigma|<\frac{\pi}{2}(1+b^2)$.

$$\begin{aligned}\Psi_\sigma^{D2} = \delta_{n',0}\nu_b^{j+\frac{1}{2}}\frac{k^{\frac{1}{4}}b^{-\frac{1}{2}}}{2\pi}&\Gamma(2j+1)\Gamma(1+b^2(2j+1))\\ &\times\left(\frac{\Gamma(-j+\frac{kw}{2})}{\Gamma(j+1+\frac{kw}{2})}e^{i\sigma(2j+1)}+\frac{\Gamma(-j-\frac{kw}{2})}{\Gamma(j+1-\frac{kw}{2})}\,e^{-i\sigma(2j+1)}\right) . \quad (4.3)\end{aligned}$$

New discrete branes can be obtained in $SL(2,\mathbb{R})/U(1)$ from the discrete AdS_2 branes in H_3^+. Like the original AdS_2 branes, the discrete AdS_2 branes give rise to two families of D-branes in the coset. Their one-point functions can be obtained from D1- and D2-branes' one-point functions thanks to the formula (3.4). Let me first consider the D1^d-branes obtained from the D1-branes:

$$\begin{aligned}\Psi^{\mathrm{D1}^d}_{(m,n)} = \delta_{w,0}e^{in'(\theta_0+\frac{\pi}{2})}\nu_b^{j+\frac{1}{2}}2k^{-\frac{1}{4}}b^{-\frac{1}{2}}&\frac{\Gamma(2j+1)\Gamma(1+b^2(2j+1))}{\Gamma(j+1+\frac{n'}{2})\Gamma(j+1-\frac{n'}{2})}\\ \times\sin\pi&\left[(2j+1)(m-\frac{1}{2})+\frac{n'}{2}\right]\sin\pi nb^2(2j+1) . \quad (4.4)\end{aligned}$$

The spectrum encoded in the annulus amplitude $Z^{\mathrm{D1}^d}_{(m_1,n_1)(m_2,n_2)}$ contains a finite number of discrete representations with positive integer multiplicities and is therefore consistent. (However, I did not find the marginal field which might have been expected from the existence of a modulus θ_0.) Note also that the amplitude $Z^{\mathrm{D1}-\mathrm{D1}^d}_{r\neq 0,(m,n)}$ suffers from the same imaginary spectrum pathology as the amplitude $Z^{AdS_2-AdS_2^d}_{r\neq 0,(m,n)}$ in H_3^+.

The $\mathrm{D2}^d$-branes obtained from the D2-branes are characterized by the one-point function:

$$\Psi^{\mathrm{D2}^d}_{(m,n)} = \delta_{n',0}\nu_b^{j+\frac{1}{2}} i \frac{k^{\frac{1}{4}}b^{-\frac{1}{2}}}{\pi}\Gamma(2j+1)\Gamma(1+b^2(2j+1))\sin\pi n b^2(2j+1) \times \left(\frac{\Gamma(-j+\frac{kw}{2})}{\Gamma(j+1+\frac{kw}{2})}e^{i\pi(m-\frac{1}{2})(2j+1)} - \frac{\Gamma(-j-\frac{kw}{2})}{\Gamma(j+1-\frac{kw}{2})}\,e^{-i\pi(m-\frac{1}{2})(2j+1)}\right) . \quad (4.5)$$

The spectrum encoded in the annulus amplitude $Z^{\mathrm{D2}^d}_{(m_1,n_1)(m_2,n_2)}$ contains a finite number of discrete representations with positive integer multiplicities and is therefore consistent. [4]

The spectrum $Z^{\mathrm{D2}-\mathrm{D2}^d}_{\sigma,(m,n)}$ is also consistent and free from the imaginary spectrum pathology, because the D2-brane parameter σ comes from pure imaginary values of the AdS_2 brane parameter r. However, it might be more relevant to examine the amplitude $Z^{\mathrm{D1}-\mathrm{D2}^d}_{r,(m,n)}$, which is more difficult to compute because of the difference in gluing conditions between D1- and $\mathrm{D2}^d$-branes. This difficulty is no obstacle to finding that $Z^{\mathrm{D1}-\mathrm{D2}^d}_{\neq 0,(m,n)}$ has the imaginary spectrum pathology [5] except if $(m,n) = (1,1)$, like the amplitude $Z^{FZZT-ZZ}_{s,(m,n)}$ in Liouville theory. Notice that $Z^{\mathrm{D1}-\mathrm{D0}}_{r\neq 0,n}$ is also free from the imaginary spectrum pathology only for $n=1$. The D0- and $\mathrm{D2}^d$-branes with parameters n and $(1,n)$ respectively behave identically in this respect because their overlaps with D1-branes only involve closed strings with winding zero, which make no difference between them: $\Psi^{\mathrm{DO}}_n(w=0) = \Psi^{\mathrm{D2}^d}_{(1,n)}(w=0)$.

4.2 Geometric and non-geometric D-branes

Let me discuss whether the new $\mathrm{D1}^d$- and $\mathrm{D2}^d$-branes have a geometric interpretation. A geometric description of the 2d black hole is possible in the limit $k\to\infty$ which corresponds to small string length $\ell_s = \sqrt{\alpha'}$ (while $\sqrt{k\alpha'}$ is a fixed length). First recall the geometric interpretation of the known D0-, D1- and D2-branes [10] as zero-, one- and two-dimensional geometric objects in the 2d black hole. This can be seen in the large k behaviour of the one-point functions,

$$\Psi^{\mathrm{D0}}_n \sim k^{-\frac{1}{2}} \quad , \quad \Psi^{\mathrm{D1}}_{(r,\theta_0)} \sim 1 \quad , \quad \Psi^{\mathrm{D2}}_\sigma \sim k^{\frac{1}{2}}. \quad (4.6)$$

How this behaviour depends on the dimensionality of the D-branes is indeed consistent with the dependence of the D-branes' tensions $T \propto (\alpha')^{-\frac{p}{2}}$ with respect to the D-branes' dimensions p.

Now the observation (from the previous subsection) that closed strings with zero winding couple identically to D0-branes and to $\mathrm{D2}^d$-branes implies that the $\mathrm{D2}^d$-branes should be interpreted as pointlike branes at the tip of the cigar like the D0-branes. The behaviour

[4] The detailed computation of this spectrum for $m_1 \neq m_2$ would require a non-trivial generalization of the calculations in [10]. Note also that the multiplicities are positive in contrast to the D2-brane case [24], due to the sign difference between the second lines of eqs (4.3) and (4.5).

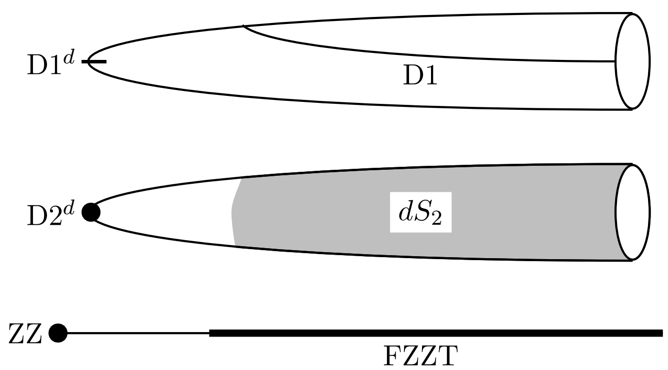
[5] The imaginary spectrum pathology is absent from such a discrete annulus amplitude if and only if $\Psi^{\mathrm{D1}}_r \left(\Psi^{\mathrm{D2}^d}_{(m,n)}\right)^*$ is a linear combination of a finite number of terms of the type $\cos\lambda(2j+1)$ with λ either real or pure imaginary. The pathology results from such terms with a generic complex λ.

of D1^d-branes is different:

$$\Psi^{\mathrm{D1}^d}_{(m,n)} \sim k^{-1}\,, \tag{4.7}$$

thus their one-point functions decrease too fast at large k to allow a geometric interpretation. It is possible to call the D1^d-branes "anisotropic localized branes at the tip of the cigar" only in a heuristic sense.

Let me nevertheless compare this heuristic geometric picture to the situation in Liouville theory. The localization of the D1^d-branes at the tip of the cigar, and the existence of continuous D1-branes extending from infinity up to some finite distance from the tip (a distance determined by their parameter r), are similar to the localization of the ZZ branes at strong Liouville coupling, together with the existence of FZZT branes extending to infinity. The situation of the D2^d- and D2-branes is quite different since the D2-branes extend to the tip where the D2^d-branes are located. However, a species of branes with the same gluing conditions as the D2-branes and a behaviour similar to that of the FZZT branes has been predicted to exist [25, 26]: the D-branes descending from dS_2 branes in AdS_3 [27], which I will also call dS_2 branes:



The geometry of the dS_2 branes in AdS_3 suggests that the dS_2 branes in the cigar are parametrized by a real number r' related to the D2-brane's parameter σ by $\sigma = \frac{\pi}{2} + ir'$. The identifications $\sigma = ir$ and $r = 2\pi bs - i\frac{\pi}{2}$ (from eq. (2.8)) then imply the relation $r' = 2\pi bs$ between the dS_2 brane parameter r' and the FZZT brane parameter s. In addition, the dS_2 brane is expected to be invariant under $r' \to -r'$ like the FZZT brane under $s \to -s$. This supports the idea of a close relationship between dS_2 branes in the cigar and FZZT branes in Liouville theory.

4.3 A shift equation from $N = 2$ Liouville theory

Let me now compare the D-branes in the 2d black hole with D-branes in the $N = 2$ supersymmetric Liouville theory. The $N = 2$ Liouville theory is indeed equivalent to the $N = 2$ supersymmetric 2d black hole theory [28], which is itself very similar to the bosonic 2d black hole theory which has been considered in this section. (On the other hand, the $N = 2$ Liouville theory is considerably more complicated than bosonic Liouville theory.) The comparison of D-branes is relevant to this article because it will provide an independent shift equation for the one-point functions of the new D1^d-branes. This is based on the article on $N = 2$ Liouville theory by Hosomichi [11], which among many interesting results formulates a shift equation with j-shift by $\frac{k}{2}$ in addition to the shift equation with j-shift by $\frac{1}{2}$ considered in subsection 3.2. These two possible shifts are independent if k is not rational. However, in contrast to the two elementary α-shifts in Liouville theory (by $\frac{1}{2b}$ and $\frac{b}{2}$) which are related by a simple selfduality of the theory, the two shifts in N=2 Liouville theory must be analyzed independently.

The D1-branes in the 2d black hole (4.2) correspond to Hosomichi's *B-branes*, [11] (4.55). According to the principles of subsection 3.2, the $\mathrm{D1}^d$-branes should therefore satisfy

$$\frac{\Psi^{\mathrm{D1}^d}_{(m,n)}(j=-\frac{k}{2},n')}{\Psi^{\mathrm{D1}^d}_{(m,n)}(j=0,n'=0)} \overset{!}{=} c^{\updownarrow}t^{\updownarrow}\left(\frac{n'}{2},-\frac{n'}{2}\right)\Big|_{r=i\pi(m-\frac{1}{2}\pm nb^2)} , \tag{4.8}$$

where $c^{\updownarrow}t^{\updownarrow}$ is explicitly known [11] (4.55), and the $N = 2$ Liouville degenerate spin $\frac{k}{2}$ becomes $-\frac{k}{2}$ in $SL(2,\mathbb{R})/U(1)$ after $k \to k-2$ and reflection. If proper care is taken of the other differences of conventions, this equation is found to hold.

The D2-branes in the 2d black hole (4.3) correspond to Hosomichi's *chiral or anti-chiral A-branes*, [11] (3.26). The $\frac{k}{2}$-shift equation for these branes [11] (4.33) has a vanishing left-hand side, leading to the condition:

$$\frac{\Psi^{\mathrm{D2}^d}_{(m,n)}(j=-\frac{k}{2},w)}{\Psi^{\mathrm{D2}^d}_{(m,n)}(j=0,w=0)} \overset{!}{=} 0. \tag{4.9}$$

Surprisingly, this equation holds due to the denominator being infinite. It therefore provides a rather trivial check of the discrete D2-branes's one-point function $\Psi^{\mathrm{D2}^d}_{(m,n)}$.

To summarize, translating the new AdS_2^d branes to the $\mathrm{D1}^d$-branes in the 2d black hole and then to $N = 2$ Liouville theory has yielded a strong independent check of their consistency.

In addition, the new AdS_2^d branes translate into two new families of discrete D-branes in N=2 Liouville theory, associated to the continuous *B-branes* and *chiral or anti-chiral A-branes* of [11]. Note in particular that the N=2 Liouville incarnation of the $\mathrm{D2}^d$-branes differ from the already known *non-chiral degenerate A-branes*, [11] (3.23). These discrete A-branes are actually associated to the continuous *non-chiral non-degenerate A-branes*, [11] (3.21). Since there exist two types of continuous A-branes in N=2 Liouville theory (chiral or anti-chiral on the one hand, non-chiral on the other hand), it is not surprising that there exist two corresponding types of discrete A-branes.

For completeness, let me point out that the *degenerate chiral A-branes*, [11] (3.33) and their special case the *identity A-brane*, [11] (3.18) clearly correspond to D0-branes in the 2d black hole. It would be interesting to study the completeness of D-branes in the 2d black hole and in N=2 Liouville theory.

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